3

HYDRODYNAMICS

3.1 INTRODUCTION

Around 70% of the earth's surface is covered by water, with 97% of this water can be found in the oceans. Depths can vary from 11 km to just a few millimetres at the coastline. Similarly, various physical phenomena that occur in the sea also have a wide range of length scales associated with them. At the planetary scale, one can find tides and large-scale currents driven by Coriolis forces to the Kolmogorov Scale, where energy dissipation occurs.

For an offshore engineer, the most relevant scales are those of the same order of magnitude as typical offshore structures such as jackets, Floating Production Storage and Offloading (FPSO) and other floating structures. Those which are in the scale of hundreds of meters, the most relevant ones are ocean waves, currents and internal waves that occur on pycnoclines. This phenomenon will be described later in this chapter. The focus will be on predicting the best characteristics of these phenomena from the perspective of an offshore engineer.

3.2 OCEAN WAVES

One can typically start with regular waves in the description of ocean waves and add irregular waves as a superposition of large regular waves.

3.2.1 Regular Waves

The defining characteristic of regular waves propagating over the surface of a homogenous fluid and a flat impermeable bed is that they propagate without having any change of form. Sinusoidal trigonometric functions are the transcendental mathematical functions that can approximate this surface deformation. The reason regular waves propagate as sinusoidal waves without changing their form is the fact that these waves comprise a single frequency, f, of the time period, $T = \frac{1}{\mathrm{f},}$. It is normal to express the frequency as circular frequency,

$\omega = 2\pi f$ for these regular waves. The speed at which a regular wave propagates without change in its form is known as the phase speed, c. If any point on the surface is followed by the propagation of the wave, this point will move at a speed c. The distance over which a single cycle gets completed in space is known as the wavelength, λ. The quantity equivalent to ω in space is known as the wavenumber, $k = 2\pi/\lambda$. The height, H is known as the distance in the vertical direction of the wave between the highest point of the deformed surface (crests) and the lowest point (troughs).

A depictive sketch that has marked all the physical length parameters described so far is provided in Figure 3.1. The origin is considered to be placed at the still water level, i.e., the water surface if no disturbances are present, with x coordinate pointing in the direction of propagation of the waves while z coordinate is pointing upwards. The flat impermeable bed is at distance $z = -d$ in this coordinate system, where d is the constant water depth where the still water level is $z = 0$. Also, the sinusoidal nature means that the crests and troughs are at an equal distance from the still water level, shown as $a = H/2$, known as the wave's amplitude. The surface of the envelope of sinusoidal disturbance that is propagating without any change is given by the function η. Since the wave is propagating forward in time and at the same time is varying over distance x, η can be considered as a function of both x and t. A functional form that describes this surface elevation is given by Equation 3.1 as,

$$\eta(x, t) = a \sin(\omega t - kx) \tag{3.1}$$

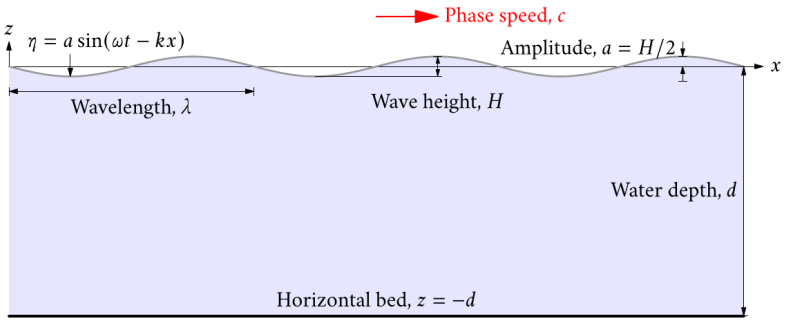

Figure 3.1 Definition sketch for regular waves propagating on the surface of a constant density fluid bounded by a flat impermeable bed

Based upon this formula, the surface elevation depicted in Figure 3.1 corresponds to the time instance at $t = 0$ as $\eta(x = 0, t = 0) = 0$ and also, the form of the surface elevation is $\eta(x, t = 0) = a \sin(\omega \times 0 - kx)x = a \sin(-kx)$. Similarly, if one picks the point $x = 0$ and plots the surface elevation time history at this point, it will be given by $\eta(x = 0, t) = a \sin(\omega t)$. Figure 3.2 provides a plot of $\eta(x)$ for three different time instances $t_3 > t_2 > t_1 = 0$. The surface elevation time history at $x = 0$ is also plotted, and the three-time points are marked.